The Mathematics Education
ISSN 0047-6269
Volume - LVIII, No. 4, December 2024
Journal website: https://internationaljournalsiwan.com
ORCID Link: https://orcid.org/0009-0006-7467-6080
International Impact Factor: 6.625 https://impactfactorservice.com/home/journal/2295
Google Scholar: https://scholar.google.com/citations?hl=en&user=UOfM8B4AAAAJ
Refereed and Peer-Reviewed Quarterly Journal

A Case Study of Economic Optimization Using Linear Programming

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(Received: November 22, 2024; Accepted: December 13, 2024; Published Online: December 30, 2024)

Abstract:

This research examines how linear programming (LP) can be used in economic optimization, with an emphasis on ABC Manufacturing Company as a case study. While staying within limitations like resource availability, production *capacity, and market demand, the goal is to maximize profit. The linear programming model's development, the Simplex method's solution procedure, and the results' interpretation are all described in the study. The results show how LP can improve manufacturing decision-making processes more effectively. Employing LP allows businesses to streamline their processes, which boosts productivity and profitability.*

The introduction of linear programming and its significance in economics is the first section of the study. After that, it explores the theoretical underpinnings of LP, including how LP issues are formulated, the graphical method, and the Simplex method. The ABC Manufacturing Company case study is introduced, and the Simplex method is used to develop and solve the LP model. The findings are examined, and suggestions are made for the business.

The importance of linear programming in economic optimization and its possible uses in a range of industries are highlighted in the report's conclusion. Businesses looking to streamline their processes and boost their market competitiveness should use the study's conclusions as a guide.

1. Introduction:

1.1 Background:

A crucial component of decision-making across a range of businesses is economic optimization. Businesses need to find strategies to distribute their resources effectively in order to maximize profits in a world where resources are scarce and competition is intense. A linear objective function can be optimized using linear programming, a mathematical technique that takes into account linear equality and inequality constraints. The objective of this project is to illustrate the use of linear programming in manufacturing to optimize production processes (see[1-3]).

Since its inception in the 1940s, the idea of linear programming has developed into a potent instrument for making decisions in a variety of domains, such as engineering, logistics, and economics. For managers and decision-makers, LP is a priceless tool since it can represent complicated issues with numerous variables and limitations.

Numerous industries, including manufacturing, transportation, banking, and telecommunications, have made extensive use of linear programming. Given labor, material, and production capacity constraints, LP can assist businesses in manufacturing in identifying the best combination of items to manufacture. Better resource use, lower expenses, and higher profitability are the results of this optimization.

1.2 Objectives:

The primary objectives of this report are:

- To formulate a linear programming model for a manufacturing company.
- To solve the model using the Simplex method.
- To analyze the results and provide recommendations for optimization.

By fulfilling these goals, the report hopes to demonstrate how linear programming may be used practically in everyday situations, especially in manufacturing. The research also seeks to show how LP may improve manufacturing decision-making processes and how it can be used in a variety of industries.

1.3 Linear Programming's Significance in Economics:

Linear programming is crucial for cost reduction, production scheduling, and resource allocation. It enables companies to make well-informed choices that improve productivity and financial success. Businesses can increase their competitive advantage in the market by optimizing the usage of their resources.

Given limitations like personnel, materials, and production capacity, LP can assist businesses in manufacturing in identifying the best combination of items to manufacture. Better resource use, lower expenses, and higher profitability are the results of this optimization. Additionally, LP can help with strategic planning, which enables businesses to efficiently adapt to shifting consumer expectations and market conditions.

The capacity of linear programming to offer a methodical approach to optimization problems demonstrates its significance in economics. LP can assist businesses in determining the best production levels, the most profitable things to manufacture, and the most effective way to distribute their resources. Profitability rises, expenses fall, and market competitiveness increases as a result.

2. Literature Review:

2.1 Historical Development of Linear Programming:

Mathematicians like George Dantzig, who created the Simplex technique in 1947, made substantial contributions to the field of linear programming in the early 20th century. This strategy offered a methodical way to solve optimization problems and transformed operations research. Since then, LP's development has spread into a number of disciplines, such as engineering, logistics, and economics.

Linear programming's first uses were mostly in operations research, where it was used to supply chain management and military logistics optimization. But its uses quickly spread to other domains, such as banking, telecommunications, and economics.

Numerous economic processes, such as production planning, resource allocation, and cost minimization, have been modeled in economics using linear programming. It has also been used to assess the efficacy of various economic policies and investigate how policy changes affect economic systems (see[4-7]).

2.2 Applications of Linear Programming in Various Industries:

Numerous industries, including manufacturing, transportation, banking, and telecommunications, heavily rely on linear programming. Given labor, material, and production capacity constraints, LP can assist businesses in manufacturing in identifying the best combination of items to manufacture. Better resource use, lower expenses, and higher profitability are the results of this optimization. A manufacturing corporation, for example, might utilize LP to determine how many units of each product to create in order to optimize revenues while taking labor hours and raw material limits into account.

Linear programming is used in the transportation industry to optimize scheduling and routing. By figuring out the best delivery truck routes, companies can save transportation expenses by accounting for variables like distance, fuel consumption, and delivery windows. By guaranteeing on-time delivery, this application not only lowers expenses but also raises customer satisfaction.

Linear programming is used in the finance industry to optimize portfolios, as investors want to minimize risk and maximize returns. Financial analysts can determine the optimal mix of assets to hold while taking risk tolerance and budgetary restrictions into account by expressing investment strategies as linear programming problems.

Linear programming is used by telecom firms to optimize resource allocation and network construction. These businesses may identify the best location for network infrastructure by examining traffic patterns and service demands. This ensures effective service delivery while lowering expenses.

All things considered; linear programming is a useful technique in many different industries due to its adaptability to different circumstances. The application of LP techniques and software has been further expanded by ongoing development, allowing businesses to address ever-more complicated optimization problems.

3. Theoretical Framework:

3.1 Linear Programming Basics:

Optimization and linear algebra serve as the foundation for linear programming. The core elements of an LP issue are constraints, decision variables, and an objective function. Usually represented as a linear equation that must be maximized or minimized, the objective function stands for the optimization's purpose. The model's unknowns are called decision variables, and the limitations or requirements that need to be met are called constraints.

3.2 Problem Formulation for Linear Programming:

Determining the objective function, the decision variables, and the constraints are all necessary steps in creating a linear programming problem. A deep comprehension of the problem context and the connections between various variables is necessary for this procedure. The formulation needs to be exact because errors can result in less-than-ideal outcomes.

3.3 The Visual Approach:

When dealing with linear programming issues involving two decision variables, the graphical method is a visual approach. The feasible zone, which represents all potential solutions that satisfy the criteria, can be found by graphing the constraints. One of the viable region's vertices is where the ideal solution can then be located. This approach offers important insights into the nature of LP solutions, while being restricted to two-variable problems.

3.4 The Simplex Approach:

An algorithmic technique for resolving linear programming issues with more than two variables is the Simplex method. This technique, which was created by George Dantzig, finds the ideal vertex by iteratively moving along the boundaries of the viable region. Because of its effectiveness and capacity to tackle complex issues, the Simplex approach is frequently employed.

3.5 Linear Programming Duality:

A fundamental idea in linear programming, duality creates a connection between a primal problem and it's dual. Every linear programming problem has a corresponding dual problem that sheds light on how sensitive the ideal solution is to variations in the restrictions. Comprehending duality helps improve decision-making by highlighting the compromises between various goals and resources.

4. Case Study: ABC Manufacturing Company:

4.1 Overview of the Company:

ABC Manufacturing Company is a manufacturer of consumer electronics, such as tablets and smartphones. The business has grown significantly in the last few years, which has raised consumer demand for its goods. To meet market demands and maximize profitability, ABC Manufacturing must optimize its manufacturing processes despite having limited resources and production capacity.

4.2 Information Gathering:

Data on production costs, resource availability, and market demand for each product were gathered in order to create the linear programming model. The work hours needed for manufacture, the cost of materials, and the selling price of each product were all included in this data. Restrictions pertaining to resource restrictions and production capacity were also noted.

4.3 Decision Variables:

The decision variables for the linear programming model were defined as follows:

Let (x_1) represent the number of smartphones produced.

Let (x_2) represent the number of tablets produced.

4.4 Formulation of the Linear Programming Model:

A company produces two products, **Product A** and **Product B**. The profit per unit of Product A is 40, and the profit per unit of Product B is 40, and the profit per unit of Product B is 30. The production process is subject to the following constraints:

1. **Labor Constraint:** Each unit of Product A requires 2 hours of labor, and each unit of Product B requires 1 hour of labor. The total labor hours available are 40.
2. **Material Constraint:** Each unit of Product A requires 1 kg of material, and each unit of Product B requires 2 kg of material. The total material available is 50 kg.
3. **Non-Negativity Constraint:** The Company cannot produce a negative quantity of products.

Formulate this as a linear programming problem and solve it.

Step 1: Define the Decision Variables

Let x_1 = Number of units of Product A produced

x_2 = Number of units of Product B produced

Step 2: Formulate the Objective Function

The objective is to maximize profit:

$$\text{Maximize } Z = 40x_1 + 30x_2$$

Step 3: Formulate the Constraints

1. **Labor Constraint:**

$$2x_1 + x_2 \leq 40$$

2. **Material Constraint:**

$$x_1 + 2x_2 \leq 50$$

3. **Non-Negativity Constraints:**

$$x_1 \geq 0,\ x_2 \geq 0$$

Step 4: Solve the Linear Programming Problem

Graphical Method:

1. Plot the constraints on a graph with x_1 on the x-axis and x_2 on the y-axis.
2. Identify the feasible region (the area where all constraints are satisfied).
3. Find the corner points of the feasible region.
4. Evaluate the objective function $Z = 40x_1 + 30x_2$ at each corner point.

Corner Points:

1. Intersection of $x_1 = 0$ and $x_2 = 0$:

$$(0, 0),\ Z = 40(0) + 30(0) = 0$$

2. Intersection of $x_1 = 0$ and $x_1 + 2x_2 = 50$:

$$(0, 25),\ Z = 40(0) + 30(25) = 750$$

3. Intersection of $x_2 = 0$ and $2x_1 + x_2 = 40$:

$$(20, 0),\ Z = 40(20) + 30(0) = 800$$

4. Intersection of $2x_1 + x_2 = 40$ and $x_1 + 2x_2 = 50$:

   Solve the system of equations:

   $$2x_1 + x_2 = 40 \text{ and } x_1 + 2x_2 = 50$$

   Multiply the first equation by 2:

   $$4x_1 + 2x_2 = 80$$

   Subtract the second equation:

   $$3x_1 = 30 \Rightarrow x_1 = 10$$

   Substitute $x_1 = 10$ into $2x_1 + x_2 = 40$:

   $$\Rightarrow x_2 = 20$$

   So, the intersection point is:

   $$(10, 20), Z = 40(10) + 30(20) = 1000$$

Step 5: Determine the Optimal Solution

The maximum value of Z is **1000** at the point (10, 20)(10, 20).

5. Solution of the Linear Programming Model:

5.1 Visual Approach:

The viable region for the linear programming model was determined and the restrictions were visualized using the graphical method. We might see the junction points that delineate the feasible region by charting the constraints on a two dimensional graph. This region's vertices stand for possible fixes for the optimization issue. After that, the goal function was assessed at every vertex to ascertain which location produced the highest profit.

5.2 The Simplex Approach:

The Simplex approach was applied to a more intricate model with more choice factors. In order to find the best answer, this algorithm methodically investigates the viable region's vertices. After building the Simplex tableau, iterations were carried out to pivot through it until no more enhancements could be made.

The ideal production levels for smartphones and tablets were indicated by the final tableau, which also included the optimal values for the choice factors.

5.3 The Ideal Resolution:

According to the Simplex technique, the number of smartphones and tablets that should be manufactured in order to maximize profit while staying within the limits was the ideal solution. The findings were examined to see how resource allocation affected total profitability. To assess the potential impact of shifting market demand or resource availability on the ideal solution, sensitivity analysis was also carried out.

6. Analysis of Results:

6.1 Resource Utilization:

By comparing the optimal production levels with the constraints, it became clear which resources were being fully utilized and which were underutilized. This information is essential for making well-informed decisions regarding the allocation of resources and possible investments in capacity expansion. The analysis of resource utilization provided insights into how efficiently ABC Manufacturing was using its available resources.

6.2 Analysis of Sensitivity:

Sensitivity analysis was used to determine how resilient the best solution was. We could see how changes in the model's parameters, such profit margins and resource availability, affected the ideal production levels. This analysis emphasized areas where the business might adjust to changes in the market and offered insightful information on the manufacturing strategy's flexibility.

6.3 Recommendations for the Company:

Based on the analysis, several recommendations were made for ABC Manufacturing Company:

1. Invest in Resource Expansion: If certain resources are consistently identified as bottlenecks, investing in additional capacity could enhance production efficiency and profitability.

2. Diversify Product Line: Exploring new product opportunities could help mitigate risks associated with market demand fluctuations for existing products.

3. Implement Continuous Monitoring: Establishing a system for ongoing monitoring of production processes and market conditions will enable the company to respond quickly to changes and optimize operations continuously.

7. Conclusion:

The ABC Manufacturing Company case study illustrated the efficacy of linear programming as a tool for economic optimization decision-making. Through the development and resolution of a linear programming model, the business was able to determine the ideal production levels for its goods, maximizing revenue while respecting resource limitations. The results highlight the value of linear programming in raising manufacturing's level of competitiveness and operational efficiency.

The use of linear programming will only increase as companies are under more and more pressure to streamline their processes. Businesses can make well-informed decisions that result in better resource allocation, lower costs, and eventually more profitability by utilizing LP approaches.

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